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# Holographic Theory of Accelerated Observers, the S-matrix, and the Emergence of Effective Field Theory



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**Abstract**

We present a theory of accelerated observers in the formalism of holographic space time, and show how to define the analog of the Unruh effect for a one parameter set of accelerated observers in a causal diamond in Minkowski space. The key fact is that the formalism splits the degrees of freedom in a large causal diamond into particles and excitations on the horizon. The latter form a large heat bath for the particles, and different Hamiltonians, describing a one parameter family of accelerated trajectories, have different couplings to the bath. We argue that for a large but finite causal diamond the Hamiltonian describing a geodesic observer has a residual coupling to the bath and that the effect of the bath is finite over the long time interval in the diamond. We find general forms of the Hamiltonian, which guarantee that the horizon degrees of freedom will decouple in the limit of large diamonds, leaving over a unitary evolution operator for particles, with an asymptotically conserved energy. That operator converges to the S-matrix in the infinite diamond limit. The S-matrix thus arises from integrating out the horizon degrees of freedom, in a manner reminiscent of, but distinct from, Matrix Theory. We note that this model for the S-matrix implies that Quantum Gravity, as opposed to quantum field theory, has a natural adiabatic switching off of the interactions. We argue that imposing Lorentz invariance on the S-matrix is natural, and guarantees super-Poincare invariance in the HST formalism. Spatial translation invariance is seen to be the residuum of the consistency conditions of HST.

# Contents



# 1 Introduction

The Unruh effect is one of the more bizarre features of quantum field theory (QFT). An accelerated observer appears to see a thermal bath even in absolutely flat space-time. As usual, the bath is associated with a horizon, because a uniformly accelerated observer is only in causal contact with a portion of Minkowski space. Its Hamiltonian is a boost operator in the Lorentz group, and this has a Killing horizon on the light cone to which the observer's trajectory asymptotes. This is a straightforward consequence of QFT but when one attempts to find a self consistent description of the heat bath within the QFT formalism, trouble arises. In particular, unless one introduces arbitrary cutoffs near the horizon, the entropy per unit horizon area of the bath, in QFT, appears to be infinite. Thermodynamic considerations suggest instead a fixed entropy per Planck area. Note that this breakdown of QFT is an infrared problem, though one which is confined to short space-like distance near the horizon. Similarly, in a black hole space-time, a supported observer sees an infinite number of states of arbitrarily low energy, clustered near the horizon. Given that a black hole can be formed in collisions between either two high energy particles, or many low energy particles, one has to ask, "Where do these states come from?" . The Unruh effect suggests that the correct model of quantum gravity in a fixed causal diamond, should incorporate these states into the low energy spectrum[1]. The question of whether or not a black hole is formed in particle collisions depends on whether the interactions with the horizon DOF is negligible or not.

Holographic Space Time (HST) is an attempt to define a general theory of quantum gravity. Like QFT, it assigns a quantum operator algebra to each causal diamond in space-time, with the property that the algebra assigned to the maximal diamond in the intersection of two diamonds, $D_{1,2}$, is a tensor factor in each of the individual diamond algebras. This encodes the causal structure of space-time into the quantum algebra. In contrast to QFT,

[1]We will see that in HST the Hamiltonian in finite causal diamonds *must* be time independent. The real meaning of the phrase low energy spectrum is that the terms in the Hamiltonian coupling particle and horizon degrees of freedom are, for a geodesic observer in a large causal diamond, suppressed relative to the terms describing free particle propagation.

the diamond operator algebras in HST are generally finite dimensional matrix algebras, with the dimension of the Hilbert space on which the matrices act, equal to the exponential of one quarter the area of the holoscreen of the corresponding causal diamond, in Planck units. This is only an asymptotic equality when the dimension is large. We view it as the *definition* of an emergent space-time structure from the structure of a quantum mechanical system. The point of course is that, giving the areas for a sufficiently rich set of diamonds completely specifies the conformal factor. A Lorentzian geometry is completely determined by its causal structure and conformal factor.

In this paper, we will use the HST formalism to provide a novel understanding of the Unruh effect. The finite reservoir of states, which provides the thermal bath exposed in Unruh's calculation appear naturally in HST, and are *not* interpretable as particle states on a stretched horizon. In four dimensions, the HST variables are operator valued matrices and particle states correspond to states which are annihilated by certain off block diagonal matrix elements. Most of the states in the causal diamond live on the horizon and do not behave like particles. In the Hamiltonian appropriate for a geodesic observer all terms involving the horizon DOF are small than those describing particle propagation by a factor $1/N$ (where $N$ is the radius of the horizon in Planck units), but their Hamiltonian is a *fast scrambler* [1]. We will argue that particle interactions in HST are mediated by interactions with the horizon states, in a manner we will make precise below. For geodesic observers, in large causal diamonds, the horizon states become un-entangled with the particles. For accelerated observers the dis-entanglement is incomplete, and this gives rise to the Unruh temperature. This picture also clarifies previous discussions of dS space in HST. Throughout the paper we will neglect factors of $2\pi$ and concentrate on scaling laws. Most of these factors are conventions relating the integers in the HST construction to geometrical radii.

The biggest difference between HST and QFT lies in the structure of their Hamiltonians. A QFT has a single Hamiltonian, once a set of space-time coordinates is chosen. In HST, the same space-time is described by an infinite set of time dependent Hamiltonians, each describing physics from the point of view of proper time along a single time-like trajectory, and each operating in a different Hilbert space. Each trajectory in a congruence gives us a complete description of the physics[2], but the descriptions are redundant. The redundancy is described in terms of an overlap Hilbert space $\mathcal{O}(t, x, y)$ for every pair of trajectories $(x, y)$ and every time $t$. This Hilbert space is a tensor factor in each of the individual diamond Hilbert spaces at time $t$ , $\mathcal{H}(t, x)$ and $\mathcal{H}(t, y)$ and $\mathcal{H}(t, x)$ is a tensor factor in $\mathcal{H}(t + 1, x)$, so we can write $\mathcal{H}(t, x) = \mathcal{P}^{f(t)}$, where the *pixel Hilbert space* $\mathcal{P}$, will be described below. $f(t)$ encodes the way the area of a causal diamond in the space-time under consideration grows with proper time. In regions where spatial curvature is negligible it scales like $t^{d-2}$ in $d$ dimensional space-time. The Hamiltonians $H(t, x)$ all operate in $\mathcal{H}(t_{max}, x) \equiv \mathcal{H}(x)$, where $t_{max}$ is the time at which the Hilbert space $\mathcal{H}(t, x)$ reaches its maximum dimension, which might be infinite. The significance of the nested tensor factorization of $\mathcal{H}(t, x)$ inside $\mathcal{H}(t_{max}, x)$ is that the evolution operator $U(t, -t, x))$ factorizes into an operator on the smaller space, multiplied by an operator on its tensor complement in the maximal space.

[2]Except in situations where there are black holes. In that case, trajectories that fall into the hole generally only capture part of the full physical information on space-time. Cosmological Big Crunches also lead to observers with partial information.

The evolution operator is related to the time dependent Hamiltonian by the usual Dyson formula[3] The infinite set of quantum systems labeled by different values of $x$ are related by the consistency requirements that *the density matrix* $\rho(t,x)$ *in* $\mathcal{O}(t,x,y)$*, induced by the time evolution in* $\mathcal{H}(x)$*, is unitarily equivalent*

$$\rho(t,x) = V^{\dagger}(t,x,y)\rho(t,y)V(t,x,y),$$

*to that induced by time evolution in* $\mathcal{H}(y)$. This constrains all of the Hamiltonians, as well as the choice of initial states in the different systems and the choices of overlaps. There may be additional constraints coming from multiple overlap conditions. Our working models satisfy all of them. We consider any consistent solution of these constraints to be a quantum space-time. The constraints are extremely strong, and in our extant examples they force the geometry to be an FRW metric, with spatially flat sections near the Big Bang. Ironically, we do not yet have a complete HST description of Minkowski space, and this paper should be viewed as a small step towards achieving such a description.

The space of labels $x$ should be thought of as denoting different time-like trajectories in a congruence. The overlap Hilbert spaces determine the Lorentzian geometry of the emergent space-time. In all extant models, we take the space of labels to be a lattice with the topology of $d-1$ dimensional flat space, where $d$ is the number of non-compact space-time dimensions. One can view this as defining the topology of a Cauchy surface in the emergent space-time. The only *a priori* constraints on the geometry is that $\mathcal{O}(t,x,y) = \mathcal{P}^{f(t-1)}$ for lattice nearest neighbors and that the dimension of this space does not increase with the minimal number of lattice steps between $x$ and $y$. This makes the geometry compatible with the lattice topology.

Space-time geometry is determined by dynamics in HST but it is not a fluctuating quantum variable. Instead it is a coarse grained hydrodynamic variable, as suggested by Jacobson [3]. The true quantum variables are sections of the spinor bundle over the holoscreen [4] [5], which are the quantum versions of the orientations of pixels on the holoscreen, following the ideas of Cartan and Penrose. The holographic principle says that if the holoscreen area in Planck units is finite, then the number of sections of this bundle is finite, and they must be quantized in a finite dimensional Hilbert space. We impose the first condition by an eigenvalue cutoff on the Dirac operator on the compact Euclidean holoscreen [6]. This preserves all symmetries of the manifold, as well as covariantly constant spinors, which lead to zero modes of the Dirac operator[4]. A supersymmetric compactification is a factorization of the spinor variables into a cutoff spinor bundle over a sphere $S^{d-2}$ tensored with the spinor bundle over a compact manifold with a covariantly constant spinor. On the sphere, the eigenvalue cutoff on the Dirac Equation is equivalent to an angular momentum cutoff[5].

[3]The careful reader will have noted that our time evolution is actually discrete, so we mean the discrete analog of the Dyson formula. In principle, one can take the time step to shrink as $t$ increases, such that it scales like the inverse mass of a black hole whose radius is $t$ (we often choose a time parameter which scales like the radius of the causal diamond at time $t$). However, in this paper we will be concentrating on space-times which are rotation invariant. In order to add full angular momentum multiplets to the operator algebra in $\mathcal{H}(t,x)$ at each time step, we must take the step size to be the Planck time. Failure to do this led to a discrepancy between the quantum formalism and geometry in [2].

[4]There is a simple generalization of this picture, which gives supersymmetric flux compactifications.

[5]That is, a cutoff on the eigenvalue of the quadratic Casimir of $SO(d-1)$. For large eigenvalue this scales like the square of an integer, $L$.

We'll label the components of spinor spherical harmonics by $a, b, ...$ and a basis of sections of the internal spinor bundle by $P, Q....$ $P = 0$ labels the zero mode of the internal Dirac operator, which corresponds to the covariantly constant spinor. If there is more than one, we will get more than the minimal set of SUSY generators by the procedure we're about to outline.

The anti-commutation relations of the fundamental variables have the form

$$[(\psi_P)_a, (\psi_Q^{\dagger})_b]_+ = \delta_{ab} Z_{PQ},$$

where for each value of $a$ we have the same finite dimensional representation of a super-algebra, generated by the fermionic variables. The $Z_{PQ}$ are the bosonic generators of the algebra. They transform as sections of the bundle of forms over the manifold, and are generalizations of wrapped brane charges in string theory. Ultimately, it is the holographic principle which requires that this representation be finite dimensional.

For even $d$ the eigenvalue degeneracy of Dirac eigenvalues $\pm(L + \frac{d-2}{2})$ is

$$D_d(L) = \frac{2^{\frac{d-2}{2}}(d + L - 3)!}{L!(d-3)!},$$

while for odd $d$ we have

$$D_d(L) = \frac{2^{\frac{d-3}{2}}(d + L - 3)!}{L!(d-3)!}.$$

In both cases, the large $L$ multiplicity of the sum of harmonics up to $L$, which is the dimension of the cutoff spinor bundle, scales like $L^{d-2}$. It is given by

$$\Sigma_L = 2^{[\frac{3d-7}{2}]} \prod_{k=1}^{d-2} \frac{L+k}{k}.$$

The square brackets in the power of 2 stand for the greatest integer function. Using the Bekenstein Hawking formula, we find that $L$ is proportional to the radius of the sphere, measured in $d$ dimensional Planck units. Note that the entropy of a single spinor harmonic component is the logarithm of the dimension of the representation of the super-algebra. If the internal eigenvalue cutoff is $K$, then because, at high wave number every smooth manifold looks locally flat, the number of independent fermionic generators of the super-algebra scales like $K^D$, where $D$ is the dimension of the internal manifold. Thus, since we've chosen a representation irreducible w.r.t. the fermionic generators, the dimension of the representation scales like $e^{K^D}$ and $K^D$ scales like the volume of the internal manifold in $d + D$ dimensional Planck units. The entropy per $d$ dimensional pixel scales like the internal volume, which is the usual Kaluza-Klein scaling of the $D$ dimensional Planck mass with the internal volume.

In this paper we will restrict attention to $d = 4$, where the chiral spinor bundle consists of $N \times N + 1$ matrices $\psi_i^A(P)$. Their conjugates $\psi_B^{\dagger\, j}(P)$ transform in the anti-chiral bundle. $N$ is an angular momentum cutoff, which is also a cutoff on the Dirac eigenvalue on the 2-sphere. The maximal angular momentum is $N - \frac{1}{2}$. The commutation relations are

$$[\psi_i^A(P), \psi_B^{\dagger\, j}(Q)]_+ = \delta_i^j \delta_B^A Z_{PQ}.$$

Bilinears $\psi\psi^\dagger$ are $N \times N$ matrices. If we consider $\psi$'s constrained by the requirement that these square matrices be block diagonal, with $B$ blocks of size $N_i \times N_i$, then a Hamiltonian constructed from single traces of these bilinears will not couple the blocks, and will have a permutation symmetry that will act as statistics on the states made of single block operators. Furthermore, the statistics will be connected to spin, because the $\psi$ operators from different blocks anti-commute, and carry half integer spin.

Now we would like to show that as $N_i \to \infty$ there is more to the name particle than just statistics. As usual in this limit of a sphere of infinite size, we have to invoke invariance of the physics under a conformal group in order to obtain a sensible limit. We are interested in limits that give rise to asymptotically flat space, so the relevant conformal group is that of the 2 sphere itself (rather than a sphere times a line, which would give rise to asymptotically AdS space), which is $SO(1,3)$. In this limit, our finite basis of spinor sections becomes complete in the space of square integrable spinor sections on the sphere. We can choose a continuous "basis" of delta function sections

$$\psi\delta(\Omega - \Omega_0),$$

where $[\psi(P), \psi^\dagger(Q)]_+ = pZ_{PQ}$. For the zero mode, $Z_{00} = 1$. $p$ is a positive normalization which, appears in the transition between the discrete finite basis and the continuous one. If we take the limit block by block, the ratios of the $p$ factors are just the ratios of block sizes.

The conformal Killing spinor equation

$$D_m q^a = e_m^A (\gamma_A)^a_b p^b,$$

where $D_m$ is the covariant derivative in the spinor bundle, has a solution space which transforms as the spinor representation of $SO(1,3)$. Denote the solutions by $q^a_\alpha(\Omega)$. Integrating them against the delta function generators, we get operators $Q_\alpha(\Omega_0)$, which satisfy

$$[Q_\alpha(\Omega_0), \bar{Q}_{\dot\beta}(\Omega_0)]_+ = p(1,\Omega)_\mu \sigma^\mu,$$

where $\sigma^\mu$ are the $SO(1,3)$ Weyl matrices.

Using this formula in a large number of large blocks along a diagonal, we obtain a Fock space of massless superparticles, with arbitrary momentum. The spectrum of superparticles is determined by the non-trivial structure of the superalgebra. We require it to contain exactly one graviton multiplet, and no massless particles of higher spin.

Our conclusion is that, if the internal manifold has a covariantly constant spinor, and the sphere is taken to infinite size in a manner consistent with Lorentz invariance, then there are degrees of freedom in the system which approach those of a super-symmetric quantum field theory. On the other hand, if we count the total number of degrees of freedom then the overwhelming majority are not of this type.

In order to understand the correct counting of particles, and scaling of their momenta in a finite causal diamond, we must understand the way in which the holographic radial direction arises in Minkowski space. The pixel variables we have identified with particles, are better thought of as analogs of actual pixels on the holoscreen. They count the total momentum that comes out through a particular spherical cap. If the causal diamond has area $\sim N^2$ and the size of the block is $K$, then the solid angle subtended by the cap is $\sim \frac{4\pi}{K^2}$, which

implies that the typical transverse momentum of a particle captured by the pixel is $K/N$. However, we will see that in order to match the de Sitter temperature, we must view $K$ to be the radial momentum in Planck units. The discrepancy is resolved by noting that the total momentum in the pixel might come from a single particle with $K$ units of radial momentum, or $N$ particles with $K$ units of the minimal momentum $1/N$, or something in between. The kinematics of the pixel operators $\psi_i^A$ can encode only the total momentum which enters or exits the large causal diamond through the corresponding pixel. The difference between the possible ways that momentum is distributed in the radial direction is encoded in time delays, which is to say the energy dependence of phases in the S-matrix, and our present kinematic discussion cannot distinguish them.

The maximal number of particles describable by this formalism is $\frac{N^2}{K}$, since $N/K$ is the maximal number of spherical caps. We must have $K > N^{1/3}$ in order for the caps not to fill the sphere uniformly. In fact, since we anticipate that the formalism will produce long range forces between the particles, the angular size of the particle wave functions must shrink more rapidly than this in order for the description of freely propagating particles to be accurate as $N \to \infty$. The bulk constraint that the particle gas not form black holes indicates that $K \sim N^{1/2}$ for the maximal entropy particle state. We do not yet have a purely holographic derivation of this constraint.

We have to understand why the particle degrees of freedom are decoupled from the rest, in the limit of an infinite causal diamond. When discussing rectangular matrices like $\psi$, we will speak of them as block diagonal when the $N \times N$ bilinear $\psi\psi^\dagger$ is block diagonal. Imagine a block diagonal configuration with several small blocks of size $K_i$ and one large block of size $N - \sum K_i$, in a causal diamond of radius $N$. We define an incoming particle state by the constraint

$$\psi_i^A(P)|particle\rangle = 0$$

, for $KN + L$ matrix elements of the spinor variables, with $1 \ll K, L \ll N$. In particular, for the block diagonal configuration just described, we have $K = \sum K_i$ and $L = \sum_{i \neq j} K_i K_j$. This decomposition makes sense because the anti-commutation relations are invariant under $U(N) \times U(N+1)$ transformations on $\psi$. We are thinking of a limit in which $N \to \infty$ with $K \sim L = aN$ and $a \ll 1$. Recall that $N$ is the size of a causal diamond, so this limit is the one which the evolution operator approaches the S-matrix.

For each sector of block diagonal $\psi$'s with a fixed number of particles, we can define the Hamiltonian $P_0$ above, in terms of the direct sum of the block diagonal SUSY generators in the blocks $K_i$. We define each of those sectors by setting all off diagonal matrix elements to zero, and each state satisfying the constraint can be viewed as belonging to one of those sectors. The Hamiltonian $P_0$ breaks the unitary symmetry down to a permutation symmetry for the blocks. Note that each block diagonal sector has one large block with of order $N^2$ non-zero matrix elements. These are the *horizon states* in the causal diamond of area $\sim N^2$, and, by definition, they give no contribution to $P_0$. We will construct a Hamiltonian for geodesic observers such that the horizon states decouple from particles as $N \to \infty$. The decoupling is less pronounced for accelerated observers and leads to the Unruh temperature in the $N \to \infty$ limit. Finally, in dS space, $N$ remains finite and even the geodesic observer thermalizes with the horizon states over long enough time scales. Accelerated Minkowski observers go over to observers at fixed static coordinate in dS space.

We define the Hamiltonian for the geodesic trajectory in a large causal diamond as

$$H(N) = P_0(N) + \frac{1}{N^2}\mathrm{Tr}\ W(\psi\psi^\dagger),$$

where $W(x)$ is a polynomial of $N$ independent order[6]. Standard large $N$ counting shows that the second term is a small perturbation for large $N$. However, $N$ plays the additional role of time in these formulae.

The time evolution operator for our system is

$$U(N,-N) = T\prod_k e^{-iH(k)}.$$

The 't Hooft coupling of order $1/N$ represents an adiabatic switching off of interactions. To see this, recall that we define particle states by a constraint

$$\psi_i^A(P)|\mathrm{particle}\rangle = 0,$$

for of order $KN$ matrix elements. Let's first imagine taking $N$ to infinity with $K$ fixed and large. As a consequence of the $1/N$ in the 't Hooft coupling and the fact that $W$ is a finite order polynomial, the possible lifting of this constraint by interactions (or increase of the number of vanishing $\psi$ operators), can at most occur for a finite number. Thus, the coefficient $K$ in the formula for the number of vanishing in and out operators

$$N_{in/out} = NK + n_{in/out},$$

becomes an asymptotically conserved quantum number. The non-vanishing operators can be arranged as a large, approximately $N \times N$ block, a number of finite $N_i$ by $N_i$ blocks, and a number of vanishing off diagonal elements. The number of particle blocks, $B_{in/out}$, and the individual $N_i$, can change, but $K = \sum N_i$ is preserved. The Hamiltonian $P_0$ is defined as above, via the SUSY algebra, in each sector with a fixed number of particles. It's bilinear in the pixel variables. It is a sum of single particle Hamiltonians, each proportional to $K_i$. The conserved quantum number $K$ is thus proportional to the asymptotic particle energies. Note that, as expected in a theory of gravity, $K$ is only defined in the limit of an infinite causal diamond.

We see that as $N \to \infty$, both in the past and the future, the particle states will decouple from the horizon states, but because the interaction only switches off adiabatically, there will be a non-trivial S-matrix relating free particle states at $t = -N \to -\infty$ to free particle states at $t = N \to \infty$. As in Matrix Theory [7], the S-matrix for scattering of particles is, at a microscopic level, mediated by non-particle like degrees of freedom. Here these are identified with the horizon variables, which account for the BHFSB [8] entropy of the causal diamond. There is no sense in which bulk quantum fields appear anywhere in the HST formalism. Instead, we can calculate an S matrix. Ancient results indicate that any low energy scattering matrix involving particles can be matched to the S-matrix computed

[6] We could also add multi-trace operators, by the standard trick of introducing non-dynamical auxiliary fields. In the large $N$ limit, those auxiliary fields take on frozen classical values and the Hamiltonian effectively becomes a single trace operator.

from an effective field theory. This result depends only on Poincare invariance, cluster decomposition and unitarity. Usually, [9] one uses it to argue that the entire theory looks like a quantum field theory, to all orders in the energy expansion, suggesting that all DOF that are ignored in the QFT approximation are high energy frozen modes, in the usual sense of effective field theory. It's long been clear, from black hole physics, that this is wrong in a theory which includes gravity. The semi-classical field equations of gravitational QFT, have black hole solutions whose properties cannot be explained in terms of a standard QUEFT paradigm. The black hole has low energy excitations, responsible for its entropy, for which QUEFT gives an infinitely bad approximation (because it assigns them an infinite entropy). It has often been claimed *faute de mieux* that a correct approximation to the properties of these states is obtained by simply imposing a cutoff by looking at a stretched horizon. While it is clear that some of the thermodynamics of black holes is easily explained by this *membrane paradigm*, it is also clear that thermodynamics is, by design, a very coarse filter for theories.

HST, like Matrix Theory, asserts that the reason that the QUEFT description of the gravitational S matrix is incomplete, is the neglect of a huge set of degrees of freedom, which are not intrinsically of high energy. In Matrix Theory, the exact quantum mechanics has a conserved energy quantum number, the light front energy, and the DOF which are not particles are decoupled by a combination of two mechanisms. The non-particle DOF are off diagonal matrix elements ("W bosons") between the diagonal blocks which represent particles, and excitations within a block, which are not in the block ground state sector. The only excitation in the block ground state sector is a set of DOF representing the free motion of a superparticle. The observables are the scattering matrix elements along branches of the moduli space, parametrized by these super-particle positions in the limit of large transverse separation.

In that limit, the W bosons get infinitely large light front energy, and decouple according to standard paradigms. The intra block excitations are decoupled by the large time limit. Particle energies are of order $1/N$ and the excitations have light front energies of order 1. There is, therefore, an S-matrix that is unitary in the superparticle Fock space, which emerges as $N \to \infty$. In a few examples with high degrees of SUSY, the light front Hamiltonian is unique, and the challenge is to show that the limiting S-matrix is Lorentz invariant. In other cases, even with maximal SUSY, the simple prescription for the Hamiltonian breaks down, and one has the freedom to tune parameters in order to obtain a Lorentz invariant S-matrix.

As we have seen, HST has many similarities to Matrix Theory. They share the block diagonal description of particle states, as well as the fact that particle interactions arise from interaction with off diagonal variables. There is however no conserved energy before one takes the large $N$ limit, and the freezing out of off diagonal DOF is a constraint defining asymptotic particle states, rather than a consequence of a large energy gap. The omitted DOF in the S-matrix description, those of the large *horizon state* block, as well as the $o(KN)$ frozen off diagonal DOF, are not high energy in any sense. Furthermore, intra-block variables are *not* frozen out. Rather they describe the angular information about asymptotic particle states, which is parametrized by explicit transverse position variables in Matrix Theory.

In order to get the super-Poincare invariant spectrum we described above, we have to take each $K_i \to \infty$. We do this by setting $K_i = a_i N$, with $\sum a_i \ll 1$, but finite in the large $N$ limit. We suspect that the only way to get a finite limiting S-matrix is to require

that, as $N \to \infty$ the S matrix is invariant under the conformal group of the two sphere, which is $SO(1,3)$. Energy conservation, already built in to our formalism, will then imply momentum conservation.

In fact, we need momentum conservation in order to satisfy the fundamental constraints of HST. So far we have discussed only the causal diamonds of a single geodesic observer in Minkowski space. The rules of HST require us to describe a dense lattice of such observers, with appropriate overlap rules, and Hamiltonians satisfying the density matrix constraints. In Minkowski space we can take the congruence of detectors that define an HST model to be a dense web of geodesic observers, with Planck scale spacing. In the infinite causal diamond limit, the boundaries of the diamonds of all of these observers all approach the conformal boundary of Minkowski space, so the density matrix constraint becomes a constraint on the pure states of the system, and the S-matrices computed by all of these detectors must be the same. This is spatial translation invariance.

## 2 The Unruh Effect

In QFT, the Unruh effect is exhibited by going to an accelerated coordinate system defined by a congruence of uniformly accelerated trajectories. In Rindler coordinates

$$ds^2 = -x^2 dT^2 + dx^2 + dr^2 + r^2 d\phi^2,$$

the congruence is given by lines of fixed $x$. Note that $T$ is a dimensionless time variable. In HST, we want to describe the Hamiltonian for evolution in the proper time of the trajectory at $x = R$. The Unruh temperature for this trajectory is $\frac{1}{2\pi R}$. We consider a causal diamond whose tips are at $\pm T_0$ on this trajectory. The area of the diamond is $4\pi \sinh^2(T_0) R^2$ and the equation for the holographic screen in these coordinates is

$$(x \cosh(T_0) - R)^2 + r^2 = \sinh^2(T_0) R^2.$$

For large $T_0$ $x$ is bounded but $r$ can be as large as $e^{T_0}$.

In order to describe the Hamiltonian we must separate the horizon and particle DOF. Our usual rules for going between coordinate systems really apply only to the particles. For those, we note that

$$\frac{\partial}{\partial T} = x \sinh(T_0) \frac{\partial}{\partial z} + x \cosh(T_0) \frac{\partial}{\partial t},$$

where $t, z$ are the usual Minkowski coordinates in the $x, T$ plane. This has the form of a rescaling, times a large boost. For our accelerated observer, the rescaling is a factor of $R$. Since we boost all particles by the same amount, we expect the scattering matrix to be boost invariant. Thus, the boost affects only our description of the initial and final states. That is, if we have some collection of particles in the geodesic observer's frame, then these particles will all have very large $z$ component of momentum for large $T_0$, as viewed by the accelerated observer. The only physical effect on the particle Hamiltonian is thus a rescaling of the energy by $x$.

On the other hand, the horizon degrees of freedom should be thought of as living on a *stretched horizon*, a congruence of accelerated trajectories with very small $x$. We can view

the small coefficient $\frac{1}{N}$ in front of the order one term in the Hamiltonian, $\frac{1}{N}\mathrm{Tr}\ W(\psi\psi^{\dagger})$, as being the rescaling factor for that very small $x$. The horizon DOF lie on the Lorentz invariant holographic screen at the boundary of the causal diamond. The Lorentz transformation only reshuffles them, and the unitary invariance of this term in the Hamiltonian means that it is invariant under all such reshuffling. Thus, we expect the accelerated Hamiltonian to have the form

$$H_R = Z(R/L_P)P_0 + \frac{1}{N^2}\mathrm{Tr}\ W(\psi\psi^{\dagger}),$$

where $P_0$ is the Hamiltonian for particles seen by the geodesic observer. The particle states seen by the accelerated observer, all have a large boost relative to those seen by the geodesic observer. $Z(R/L_P)$ is $o(1/N)$ for a trajectory on the stretched horizon, where $R \sim L_P$. It goes to one as $R/L_P$ goes to $N$.

This is our model for accelerated observers. The degrees of freedom on the horizon are a huge heat bath for the particle DOF, because the horizon Hamiltonian is hypothesized to be a fast scrambler [1]. Different accelerations correspond to different couplings to that heat bath. We have argued above that when $Z = 1$ the large $N$ limit of this finite system gives zero temperature. The bath decouples from the particles. On the other hand, for $Z \sim 1/N$ it is clear that the distinction between particles and horizon DOF becomes meaningless. The coupling between the two kinds of variables becomes strong. The second term in the Hamiltonian treats them completely democratically, and the entire system is thermalized at infinite temperature (which just means that the finite system is completely mixed). Clearly, if we take $R/L_P$ to infinity as we take $N$ to infinity, we can get any intermediate temperature we like. The resulting temperature scales with $R$ and we get the Unruh effect.

# 3 The end of the universe

Observational data suggest that the universe has entered into a phase dominated by a positive cosmological constant and will approach de Sitter (dS) space in the relatively near future. dS space is the Lorentzian continuation of the Euclidean 4-sphere. Semi-classical study of dS space with radius $R$ reveals the following properties

- dS space is a thermal system with a *unique* temperature (as measured by a geodesic observer) $T = \frac{1}{2\pi R}$ and an entropy $\pi(RM_P)^2$.
- There is a maximum black hole mass in dS space. In general, Schwarzschild-dS black holes have two horizons $R_{\pm}$ related by

$$R^2 = R_-^2 + {R_+}^2 + R_+R_-,$$

$$2M = \frac{M_P^2}{R^2}[R_+R_-(R_+ + R_-)].$$

The sum of the horizon entropies is always smaller than the $M = 0$ case of empty dS space. When $R_- \ll R_+$, the entropy deficit is $2\pi RM$. This suggests the following interpretation of the dS temperature. The density matrix of the system is maximally uncertain, but the Hamiltonian $P_0$, whose eigenvalue is $M$, has an entropy deficit

$2\pi RM$ for this eigenspace. The observant reader will note the similarity of this prescription to the connection between energy and constraints on the horizon variables, which defined the conserved energy in Minkowski space in a previous section.

- Coleman-DeLuccia tunneling probabilities between two dS minima satisfy the principle of detailed balance with entropies instead of free energies. This indicates that the system is at infinite temperature and that the entropy is the log of the number of dimensions in its Hilbert space. For a subclass of potentials [10] the transition probabilities from the lowest dS minimum to negative c.c. Big Crunches are entropically suppressed. They look like low entropy fluctuations of a finite system in equilibrium at infinite temperature. Indeed, the maximal area causal diamond in the crunching region has a microscopic area and that region is plausibly modeled as a low entropy state. Again, the simplest interpretation of this result is that the density matrix in empty dS space is the maximally uncertain one, so that free energies are equal to entropies.

- Although the early and late time spatial slices in global coordinates grow to arbitrarily large size, one cannot actually send in small perturbations from the past with impunity. Most will cause a Big Crunch before the dS throat is reached. Thus, these perturbations should not be thought of as part of the theory of a stable dS space. The entropy of perturbations that do have such an interpretation is bounded by the entropy of dS space, because high entropy initial states form black holes. This is the analog of the restriction of perturbations of AdS space to be normalizable. A major difference between these two systems is that the space of states in AdS is infinite dimensional despite the boundary conditions. One can, perhaps, make an analogy between the dS case and that of a compact phase space. Classically, there are an infinite number of solutions of the equations of motion on a compact phase space, but the Hilbert space of states is always finite dimensional. The difficulty in making this analogy precise has to do with the fact that the nominal phase space is infinite dimensional. It is only the interpretation of the Bekenstein-Hawking formula as a micro-canonical entropy (the strong HP), which tells us that dS space has a finite number of states.

Our discussion of accelerated observers is easily adapted to this situation. First, we must decouple the proper time from the holoscreen radius $N/M_P$, when that radius approaches the dS radius. After that time we postulate a static Hamiltonian, which evolves the system forever and acts on a Hilbert space with dimension $e^{\pi (RM_P)^2}$[7]. For a detector traveling along a trajectory of fixed $r$ in the static coordinate system

$$ds^2 = -(1 - r^2/R^2)dt^2 + \frac{dr^2}{(1 - r^2/R^2)} + r^2 d\Omega^2,$$

$$H = Z(r/R, R/L_P)P_0 + \frac{1}{N^2}\mathrm{Tr}W(\psi\psi^\dagger, R/L_P).$$

[7] Here we construct a model of stable dS space. In [11] we argued that dS spaces with an ensemble of values of $R$ will arise in the context of holographic cosmology. They are embedded as black holes in a background $p = \rho$ universe. The collision of these black holes would disrupt the eternal static evolution described in this paper.

For $R/L_P \gg 1$, we have $Z \approx \sqrt{1 - r^2/R^2}$, while for $r \to R$, $Z$ is approximately the redshift factor for the maximally accelerated trajectory (on the stretched horizon) in Minkowski space. In the latter limit $Z \sim 1/N$. $N = RM_P$ and the proper time is allowed to be $\gg R$.

As a consequence, even the particles viewed by the geodesic observer are eventually thermalized. The dS temperature is fixed by the entropy deficit formula

$$\Delta S = KN,$$

which relates the energy to the entropy and predicts a dS temperature $T \sim R^{-1}$, if we interpret $K$ as the particle energy measured in Planck units. This fits with our ideas based on the kinematics of particle momenta in Minkowski space, if we use the rule that a single particle state formed from a block of size $K$ has typical momentum $K/N$ in maximal entropy particle states, with $N$ particles per pixel. The radius of the holographic screen is $N/M_P$. A block of size $K$ has an angular momentum cutoff $\sim K$, which corresponds to a transverse linear momentum cutoff $\frac{K}{NM_P} = \frac{K}{R}$. This is the same as the IR cutoff on particle energies implied by our rule. States with higher energy, up to $KM_P$, require more suppressed off diagonal DOF. These can be interpreted as high momentum states of single particles, multi-particle states, or black holes. The distinctions are encoded in phase shifts in the scattering matrix, which tell us about the relative times at which different bits of energy hit the pixel in question.

TB has written extensively elsewhere [12] about the way in which this model reproduces the properties of the semi-classical theory of dS space, which were listed at the beginning of this section.

# 4 On the Emergence of Effective Field Theory

The Wilsonian paradigm for emergence of quantum effective field theories (QUEFTS) has guided our thinking about quantum gravity since the subject was first imagined. Conventional field theoretic quantization of Einstein's Lagrangian and its generalizations, leads to non-renormalizable perturbation expansions for the S-matrix. The conventional response to this is to imagine that the real theory of quantum gravity has a UV cutoff at the Planck scale. Quantum fields of wavelength much larger than the Planck scale are imagined to be effective variables, which emerge from integrating out Planck scale DOF, in a roughly Wilsonian manner. The fact that space-time geometry itself is supposed to be a fluctuating quantum variable throws a conceptual monkey wrench into this point of view, but one which we have all imagined was surmountable. Indeed, in two space-time dimensions, this sort of Wilsonian approach, including fluctuating geometry, succeeds, and leads to the world sheet theory of strings.

In 4 and higher dimensions, there are manifold hints that Wilsonian ideas are NOT the proper way to think about quantum gravity. At the most primitive level, black hole physics shows that initial states containing large numbers of particles with very small sub-energies for any few particle collection, evolve to final states where quantum gravitational effects are important. The microscopic S-matrix for black hole formation and evaporation is *not* calculable within the realm of QUEFT[8]. The Covariant Entropy Bound [8] indicates that

[8]This observation falsifies the notion of *classicalization* [14]. It was pointed out in [13] that black hole

the total number of states in a causal diamond with finite area holographic screen is finite. In four dimensions, it is of order $N^2$, where $N$ is the square root of the area, in Planck units. Considerations of black hole formation indicate that at most $N^{3/2}$ of this entropy can be accounted for by particle states. We have, in HST, modeled both the particle, and residual horizon states as particular configurations of pixel variables $\psi_i^A(P)$, in a way that incorporates this counting of entropy.

In string theory, and AdS/CFT, our real evidence for the utility of QUEFT comes from a matching procedure for the S-matrix[9]. We compute S-matrix elements for low mass particles with small sub-energies $p_i \cdot p_j$, in the underlying gravitational theory, and match them to amplitudes computed in a QUEFT. In the Minkowski case, results of Mandelstam and Weinberg [15], dating back to the 1960s, show that any set of amplitudes satisfying cluster decomposition, unitarity and some analyticity properties, will have a low energy expansion that can be matched, to any order, by a QUEFT with a complicated Lagrangian. Although there have been attempts, such as String Field Theory, to derive a QUEFT from string theory in a more conventional Wilsonian manner, these fail. The Lagrangian of string field theory is defined by a non-summable perturbation series.

More insight can be gained by thinking about the computation of the S-matrix in Matrix Theory [7]. This is a model of certain asymptotically flat string compactifications in Discrete Light Cone Quantization. The basic variables are matrices, and particle DOF are defined in terms of the coefficients of the unit matrix in a set of blocks. The off-block diagonal matrices have large light front energy only when the transverse separations are large. We thus consider initial states with some number of blocks with large transverse separation, and watch their evolution to some other number of blocks with large transverse separation. These different configurations correspond to exact flat directions in the potential of the matrix quantum mechanics. When the "particles" are close to each other, the off diagonal matrix elements are no longer high energy and cannot be distinguished from the particle DOF. They mediate transitions between the different flat directions.

The observant reader will see that we have adapted a similar mechanism to our description of particle interactions in HST, where there is no conserved energy for finite causal diamonds. The upshot of this is that the restrictions on particle amplitudes in a finite causal diamond are much more severe than those in infinite Minkowski space. The number of particles, and their energies and angular separations are severely constrained, in a complicated manner. Most of the DOF in a causal diamond are not particle like and the decoupling of particles from horizon states does not come from a Born-Oppenheimer-Wilson separation of energy scales. In particular, the states describing the near horizon dynamics of black holes, and their entropy, are *not* well modeled by QUEFT with a brick wall cutoff on a stretched horizon.

There is however, another kind of effective field theory which is valid in large but finite causal diamonds in HST. In a prescient 1995 paper [3], Jacobson argued that Einstein's equations followed from the first law of thermodynamics and the Bekenstein-Hawking entropy

formation is the key mechanism for avoiding UV divergences in quantum gravity, but that does *not* mean that the classical theory is enough to understand the process. Classical black hole formation describes only the thermodynamics of the quantum system, not its microscopic dynamics.

[9]In AdS space, the sobriquet "S-matrix" means Mellin transformed correlation functions. We will employ a similar *double entendre* for the phrases "low mass particles" and "sub-energies" in the remainder of this paragraph.

formula, applied locally to the Hamiltonian of a horizon grazing Rindler observer in an arbitrary space-time. Jacobson's gravitational field is a thermodynamic object, and he argued that it should not be quantized. This argument fits perfectly with the HST description of the Unruh effect in a large diamond as well as with the HST description of space-time as arising from the dimensions of Hilbert spaces and overlaps for an infinite number of quantum systems describing the universe from the vantage point of different time-like world lines. Space-time is a non-fluctuating variable, which describes the thermodynamics of the bulk of the DOF in HST - those which live on the horizon of causal diamonds and cannot be associated with local particle physics. Jacobson's Thermodynamic Effective Field Theory (THEFT) is thus a general feature of HST for large causal diamonds, while QUEFT applies only to a limited set of amplitudes, in situations where particle and horizon DOF are decoupled. In finite causal diamonds, this means that the bulk of the entropy of the system is not described by particles, but rather by the horizon DOF.

In a sufficiently large causal diamond in flat space-time, in the absence of black holes, the Hamiltonian of a geodesic world-line does not contain strong coupling between the large number of particle DOF and the much larger number of horizon DOF. QUEFT describes the particle interactions (which are mediated by excitation and de-excitation of horizon DOF as in Matrix Theory) with better and better accuracy as the size of the diamond goes to infinity. For an accelerated observer, this decoupling is less effective, and leads, even in the infinite diamond limit, to coupling between the particles this observer sees and the thermalized system on the horizon. As the acceleration approaches Planck scale, the distinction between particles and horizon disappears and we get a thermalized soup at infinite temperature.

# 5 Conclusions

We have outlined the first steps towards a theory of quantum gravity in 4 dimensional Minkowski space, based on the holographic principle and the idea that the appropriate variables are sections of the cut-off spinor bundle on the holographic screen. These variables describe a fuzzy pixelation of the holoscreen, which produces localized pixels in the large screen limit. Subsets of the spinor variables have the commutation relations of super-translation generators. This gives rise to a Fock space of Poincare superparticles in the limit of infinite screen, but the description contains a much larger set of variables, which do not have a conventional QUEFT interpretation, even in this limit. We have argued that these are the variables responsible for the entropy of horizons, and that the Unruh effect shows us that they are present even in the absence of black holes. We have proposed Hamiltonians written in terms of the spinor pixel variables, which demonstrate both the Unruh effect, and the way in which a scattering matrix for particles can arise in the large screen limit.

These results depend on three unusual properties of the HST formalism. There is a different (time dependent) Hamiltonian for each time-like trajectory in space-time. The horizon variables decouple because of the combination of a constraint defining particle states and adiabatic switching off of interactions. Particle interactions arise as a residual effect of the interactions between particles and the horizon at finite times. We have indicated how to derive energy conservation for the S-matrix from a rather general form of Hamiltonian for the pixel variables. The HST requirement that different trajectories give equivalent descriptions

of shared information shows us that the resulting S-matrix must also be invariant under spatial translations[10]. This, combined with general lore about the way in which infinite limits are controlled, suggest that we impose Lorentz invariance on the S matrix as well. The Lorentz group is the conformal group $SO(1,3)$ of the holographic screen. We have not studied the way to do that in this paper. However, the ancient rules of S-matrix theory tell us that the number of low energy interactions consistent with this requirement is determined by the particle content of the theory and a few parameters. Furthermore, all such S-matrices are describable by QUEFT at small values of the kinematic invariants.

The problem of particle content in HST is partly the classification of super-algebras for the pixel variables, which give rise in the large diamond limit to a spectrum of super-particles including exactly one supergraviton multiplet and no particles of spin higher than two. Based on our experience with string theory, there are likely to be other constraints on models of super-Poincare invariant gravitational S matrices in four dimensions. Certainly, while symmetries constrain the structure of the low energy S matrix to a large extent, we would expect that consistent models exist only for certain values of the dimensionless parameters (*e.g.* Yang Mills couplings) that are allowed by all symmetry constraints.

Obviously, it would also be nice to have a direct calculation of simple aspects of scattering starting from the HST formalism and the pixel variables. It seems plausible that we can find Hamiltonians which do lead to a super-Poincare invariant S-matrix, but the mechanics of this is beyond us at the moment.

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